Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Paper reference **8FM0/26**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
26: Further Mechanics 2
(Part of option J)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
- Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- Good luck with your examination.

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1.

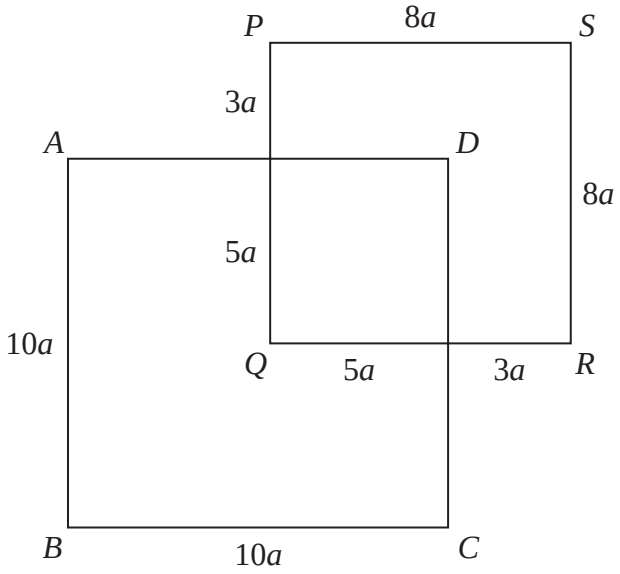


Figure 1

A uniform rod of length $72a$ is cut into pieces. The pieces are used to make two rigid squares, *ABCD* and *PQRS*, with sides of length $10a$ and $8a$ respectively. The two squares are joined to form the rigid framework shown in Figure 1.

The squares both lie in the same plane with the rod *AB* parallel to the rod *PQ*.

Given that

- *AD* cuts *PQ* in the ratio 3 : 5
- *DC* cuts *QR* in the ratio 5 : 3

(a) explain why the centre of mass of square *ABCD* is at *Q*.

(1)

(b) Find the distance of the centre of mass of the framework from *B*.

(5)

(a) Because the rod is uniform and the shape is symmetrical, the centre of mass is at the centre of the square ABCD, which is Q (1)

(b) Mass ratio of ABCD : PQRS : framework = 40 : 32 : 72 (1)
($4 \times 10a$: $8 \times 4a$: $72a$)

Horizontal distance from AB = 5a : 9a : $\bar{x}$ (1)
($\frac{10a}{2}$: $\frac{10a}{2} + \frac{5a+3a}{2}$)

Question 1 continued

$(40 \times 5a) + (32 \times 9a) = 72\bar{x}$ ← use ratios to resolve moments about AB.

$200a + 288a = 72\bar{x}$

$488a = 72\bar{x}$

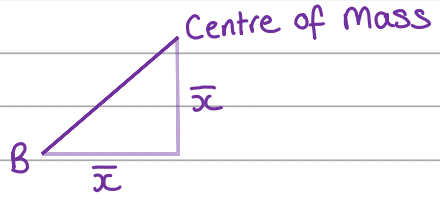
$\bar{x} = \frac{488a}{72}$

$\bar{x} = \frac{61a}{9}$ (1)

Distance $= \sqrt{\bar{x}^2 + \bar{x}^2}$

$= \sqrt{\left(\frac{61a}{9}\right)^2 + \left(\frac{61a}{9}\right)^2}$ (1) ←

$= \frac{61\sqrt{2}a}{9}$ (1)



because the framework is symmetrical

(Total for Question 1 is 6 marks)

2.

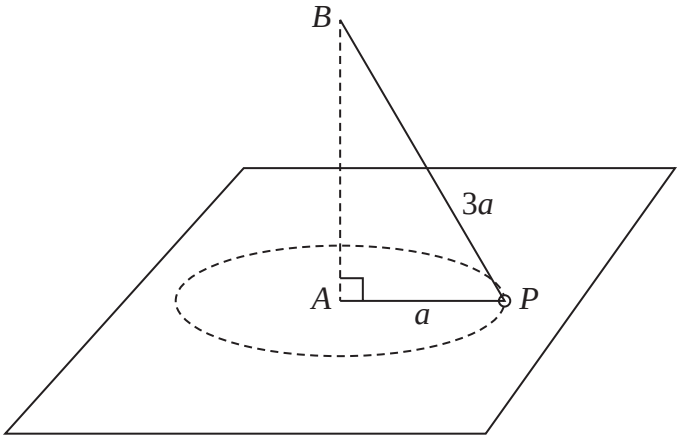


Figure 2

A small smooth ring P, of mass m, is threaded onto a light inextensible string of length $4a$.
One end of the string is attached to a fixed point A on a smooth horizontal table.
The other end of the string is attached to a fixed point B which is vertically above A.
The ring moves in a horizontal circle with centre A and radius a, as shown in Figure 2.

The ring moves with constant angular speed $\sqrt{\frac{2g}{3a}}$ about AB.

The string remains taut throughout the motion.

(a) Find, in terms of m and g, the magnitude of the normal reaction between P and the table.

(6)

The angular speed of P is now gradually increased.

(b) Find, in terms of a and g, the angular speed of P at the instant when it loses contact with the table.

(3)

(c) Explain how you have used the fact that P is smooth.

(1)

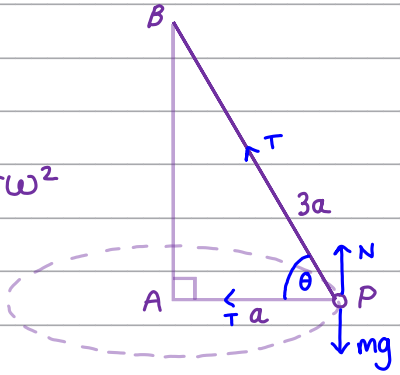
(a) Resolving vertically: ① $T\sin\theta + N = mg$ ①

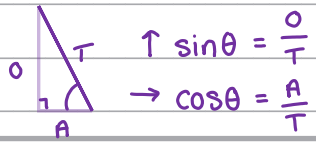
Resolving horizontally: $F = m \times a$

$a = r\omega^2$

$T + T\cos\theta = m \times r\omega^2$

$T + T\cos\theta = ma\frac{2g}{3a}$ ①





Question 2 continued

Use Pythagoras and trig to determine side lengths and θ:

$\cos\theta = \frac{a}{3a} = \frac{1}{3}$ ← $\cos = \frac{A}{H}$ ↙ $\sin = \frac{O}{H}$

$3^2 = 1^2 + opp^2 \Rightarrow opp = \sqrt{8} \Rightarrow \sin\theta = \frac{\sqrt{8}a}{3a} = \frac{\sqrt{8}}{3}$

Use exact values for $\sin\theta$ and $\cos\theta$ to form simultaneous equations. (1)

$T\sin\theta + N = mg \Rightarrow \frac{\sqrt{8}}{3}T + N = mg$ (1)

$T + T\cos\theta = ma\frac{2g}{3a} \Rightarrow T + \frac{1}{3}T = \frac{2amg}{3a}$

$\frac{4}{3}T = \frac{2mg}{3}$ ↓ $\div \frac{4}{3}$

$T = \frac{mg}{2}$ (2)

Substitute (2) into (1) and solve for N: (1)

$\frac{\sqrt{8}}{3}\left(\frac{mg}{2}\right) + N = mg$

$N = mg - \frac{\sqrt{2}mg}{3}$

$N = mg\left(1 - \frac{\sqrt{2}}{3}\right)$ (1)

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Question 2 continued

(b) Loses contact when $N = 0$:

$\updownarrow \quad T\sin\theta + 0 = mg$ ← only vertical forces on P.

$$\frac{\sqrt{8}}{3}T = mg \quad (1)$$

$$T = \frac{3mg}{\sqrt{8}} \quad (3)$$

$\leftrightarrow \quad T + T\cos\theta = ma\omega^2$

$$T\left(1 + \frac{1}{3}\right) = ma\omega^2 \quad (4)$$

$$\frac{3mg}{\sqrt{8}} \times \frac{4}{3} = ma\omega^2 \quad (1) \leftarrow \text{substitute (3) into (4)}$$

$$\frac{12mg}{3\sqrt{8}} = ma\omega^2$$

$$\frac{4g}{\sqrt{8}} = a\omega^2$$

$$\frac{4g}{\sqrt{8}a} = \omega^2$$

$$\sqrt{\frac{\sqrt{2}g}{a}} = \omega \quad (1)$$

(c) Tension in the string is the same on either side of P. (1)

Question 2 continued

(Total for Question 2 is 10 marks)

3.

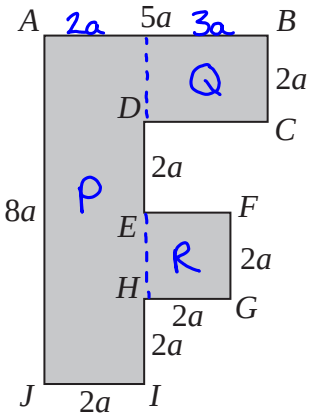


Figure 3

The uniform lamina *ABCDEFGHIJ* is shown in Figure 3.

The lamina has $AJ = 8a$, $AB = 5a$ and $BC = DE = EF = FG = GH = HI = IJ = 2a$.

All the corners are right angles.

(a) Show that the distance of the centre of mass of the lamina from *AJ* is $\frac{49}{26}a$

(5)

A light inextensible rope is attached to the lamina at *A* and another light inextensible rope is attached to the lamina at *B*. The lamina hangs in equilibrium with both ropes vertical and *AB* horizontal. The weight of the lamina is *W*.

(b) Find, in terms of *W*, the tension in the rope attached to the lamina at *B*.

(3)

The rope attached to *B* breaks and subsequently the lamina hangs freely in equilibrium, suspended from *A*.

(c) Find the size of the angle between *AJ* and the downward vertical.

(5)

(a) Note: lamina is uniform so C.o.M is in the middle of each shape.

	P :	Q :	R :	lamina	← label diagram
Mass	$2\times8=16$	$2\times3=6$	$2\times2=4$	$16+4+6=26$	(2)
→ from AJ	a	$3.5a$	$3a$	$\bar{x}$	← horizontal
→ from AB	$4a$	a	$5a$	$\bar{y}$	← vertical

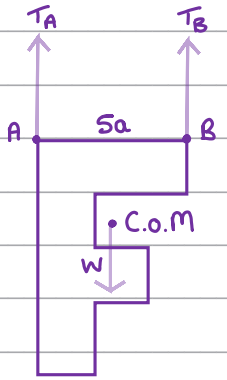
(1)

Moments about AJ: $16(a) + 6(3.5a) + 4(3a) = 26(\bar{x})$ (1)

$49a = 26\bar{x} \quad \therefore \quad \bar{x} = \frac{49}{26}a$ (1)

Question 3 continued

(b) Moments about A: $5a \times T_B = \frac{49}{26} a \times W$ (1)

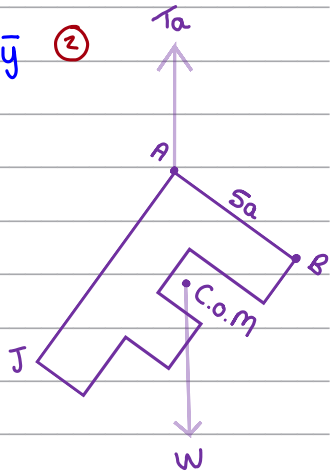


$$T_B = \frac{\frac{49}{26} aw}{5a}$$

$$T_B = \frac{49}{130} W \quad (1)$$

(c) Moments about AB: $16(4a) + 6(a) + 4(5a) = 26\bar{y}$ (2)

↑ using table in (a)



$$90a = 26\bar{y}$$

$$\bar{y} = \frac{90}{26} a \quad (1)$$

$\tan = \frac{O}{A}$

$$\tan\theta = \frac{49}{26} a \div \frac{90}{26} a$$

↑ from AJ ↑ from AB

$$\tan\theta = \frac{49}{90} \quad (1)$$

$$\theta = 28.6° \quad (1)$$

Question 3 continued

Question 3 continued

(Total for Question 3 is 13 marks)

4. A particle P moves on the x-axis. At time t seconds, $t \geqslant 0$, P is x metres from the origin O and moving with velocity $v\,\text{m s}^{-1}$ in the direction of x increasing, where

$$v = 5\sin 2t$$

When $t = 0$, $x = 1$ and P is at rest.

(a) Find the magnitude and direction of the acceleration of P at the instant when P is next at rest.

(4)

(b) Show that $1 \leqslant x \leqslant 6$

(4)

(c) Find the total time, in the first 4π seconds of the motion, for which P is more than 3 metres from O

(3)

(a) $v = 5\sin(2t) = 0$ ① ← at rest when $v = 0$

$2t = \sin^{-1}(0)$ — question states that P starts at rest (when $t = 0$) but sin

$2t = 0, \pi$ ← repeats every π, so $0 + \pi = \pi$.

$t = \frac{\pi}{2}$ ①

$\frac{d}{dv}(5\sin 2t) = 2 \times 5 \times \cos 2t$ ← $\frac{d}{dt}$ (velocity → acceleration) $\int dt$

$= 10\cos 2t$ ①

$10\cos\left(2 \times \frac{\pi}{2}\right) = -10$

∴ P moves at $10\,\text{ms}^{-2}$ towards O. ①

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Question 4 continued

(b) $\int 5\sin 2t \, dt = -\frac{5}{2}\cos 2t + c$ ①

← $\frac{d}{dt}$ (displacement → velocity) $\int dt$

When $t = 0$, $x = 1$:

$$-\frac{5}{2}\cos(2\times 0) + c = 1$$

$\cos(0) = 1$

$$-\frac{5}{2} + c = 1$$

$$c = \frac{7}{2}$$ ①

$x = \frac{7}{2} - \frac{5}{2}\cos 2t$ ① and $-1 \leq \cos 2t \leq 1$

$\Rightarrow \min(x) = \frac{7}{2} - \frac{5}{2}(1) = 1$

$\max(x) = \frac{7}{2} - \frac{5}{2}(-1) = 6$

$\therefore \quad 1 \leq x \leq 6$ ①

(c) $x = \frac{7}{2} - \frac{5}{2}\cos 2t = 3$ ← 3m from O.

$$\frac{7}{2} - 3 = \frac{5}{2}\cos 2t$$

$$\frac{1}{2} \div \frac{5}{2} = \cos 2t$$

$$\frac{1}{5} = \cos 2t$$

$$2t = 1.369..$$

$$t = 0.685 \text{ s}$$ ①

$4\pi - 8(0.685)$ ① $= 7.1$ s ①

↑

$\cos 2t$ repeats 8 times in 4π

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 11 marks)

TOTAL FOR FURTHER MECHANICS 2 IS 40 MARKS